THE SECOND INTERNATIONAL MATHEMATICAL OLYMPIAD
Blagoveshchensk – Russia, 19 March 2022

Problem 1 ***(9 points)***

Determine the number of zeros of the function

$$f(x) = 2e^{2-x^2}(x^6 - 3x^4 + 5x^2 - 1) - 2e - 5, \quad x \in R.$$

Problem 2 ***(12 points)***

Find the limit

$$\lim_{x\to 0}\left(\lim_{n\to\infty}\frac{1}{x}(A^n - E)\right),$$

where

$$A = \begin{pmatrix} 1 & \frac{x}{n} \\ -\frac{x}{n} & 1 \end{pmatrix}, \qquad E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Problem 3 ***(10 points)***

Find the sum

$$f(0) + f\left(\frac{1}{2022}\right) + f\left(\frac{2}{2022}\right) + f\left(\frac{3}{2022}\right) + \cdots + f\left(\frac{2021}{2022}\right) + f\left(\frac{2022}{2022}\right)$$

for given function

$$f(x) = \frac{a^{2x}}{a^{2x} + a}, \qquad a > 0, \ x \in R.$$

Problem 4 ***(9 points)***

Draw a line defined by the complex equation (t is a real parameter):

$$z\cdot\left(1 + e^{-it}\right)^2 = 1.$$

***Problem 5* (9 points)**

Find the limit

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} \int_1^n ln\left(1+\frac{1}{\sqrt{x}}\right) dx.$$

***Problem 6* (8 points)**

Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{n}{2^n}.$$

***Problem 7* (12 points)**

Find the volume of the m-dimensional pyramid T_m such that

$$x_1 \geq 0,\ x_2 \geq 0, \cdots, x_m \geq 0,\ x_1 +\ x_2 + \cdots + x_m \leq h.$$

***Problem 8* (12 points)**

Find real solutions of the differential equation

$$(y')^3 + \frac{2y'}{x^2} = 1 + \frac{2}{x^2} + \frac{6y}{x} + \frac{12y^2}{x^2} + \frac{8y^3}{x^3} + \frac{4y}{x^3}.$$

***Problem 9* (9 points)**

Suppose that a linear homogeneous differential equation of the order n with constant real coefficients is given. It is known that $x^{50}\sin^4(3x)$ is one of the solutions of this equation. Find the smallest possible value of n.

***Problem 10* (10 points)**

Players *A* and *B* play a chess match between themselves. Player *A* wins the game with a probability of 0.6. To even the odds, they agreed that player *A* wins if he wins three games, and *B* wins if he wins two games (draws are not counted). What is the probability of each player winning the match?